Short, Open, RC Circuit

See the course web site for due date and time

- All work must be clearly shown to receive full credit.
- Box the final answer.
- Present the homework on a single-sided and separate 8.5 x 11 inch sheets of paper in an organized, easy to read format. Do not use this homework sheet.
- Do not restate questions.
- Be sure to write your name on all pages of your homework.
- Assume all given values are accurate to at least 4 significant digits.
- Always include appropriate figure, diagrams, tables, sketches, and graphs.
- Use a fixed-pitch font (such as Courier New) for all MATLAB/Octave printouts.

RC Circuits

Example Problem
Before the switch closes, the capacitor is discharged. At time t=0 s, the switch closes. Solve for the capacitor voltage for all time.

Solution Method
1. Identify and label voltages (and currents) by redrawing the figure. Label the capacitor voltage with \( V_c(t) \) to indicate the capacitor voltage is a function of time. The polarity of the voltage is indicated with + and - symbols.

Since the capacitor is discharged before the switch closes, the capacitor voltage must be zero before time \( t=0 \) seconds.

\[ V_c(t) = 0 \text{ V, for } t < 0 \text{s} \]

3. Calculate the equivalent resistance influencing the capacitor charge or discharge.
This problem has one series resistor, therefore the equivalent resistor is just the resistor itself, namely 1k Ohm.

\[ \tau = R \cdot C \]
\[ \tau = 1 \text{ k} \Omega \cdot 1 \mu \text{F} \]
5. Determine the capacitor voltage at time $t=0$ seconds (initial voltage) and $t=\infty$ (final voltage).

The problem states that before time $t=0$ seconds, the capacitor voltage is zero. Therefore, the instant before time $t=0$, the capacitor voltage must be zero.

\[ V_c(0^-) = 0 \text{ volts} \]

Since the capacitor voltage cannot change instantaneously, the voltage an instant at which time $t=0$ seconds must also be zero volts.

\[ V_c(0) = 0 \text{ volts} \]

The capacitor is (after a long time) trying to get to 5 volts.

\[ V_c(\infty) = 5 \text{ volts} \]

6. Give the general form of the solution. Solve for $A$ and $B$ given the initial and final voltages.

\[ V_c(t) = A + B e^{-\frac{t}{RC}} \]

\[ V_c(0) = 0 = A + B e^{0} \quad V_c(\infty) = 5 = A + B e^{\frac{1}{1ms}} \]

Solve the two simultaneous equations for $A$ and $B$. Although the algebra is not shown here, you must show your algebra.

\[ A = 5 \quad B = -5 \]

7. Give the complete solution to the problem. Complete solution means for all time.

\[ V_c(t) = \begin{cases} 0 \text{ V, for } t < 0 \text{ s} \\ 5 - 5 e^{\frac{t}{1ms}} \text{ V, for } t \geq 0 \text{ s} \end{cases} \]

8. Sketch the solution for $-\tau \leq t \leq 5\tau$.

Sketches show the approximate shape of a function. Axes are labeled, with units, and the sketch is titled. Use 5 points at time $t=0$, $\tau$, $2\tau$, $3\tau$, $4\tau$, and $5\tau$ seconds to get a reasonable shape.
1. Capacitor voltage defined and labeled as $V_c(t)$.

2. Since the capacitor is discharged before the switch closes, the capacitor voltage must be zero before time $t=0$ seconds.

3. This problem has only one series resistor, therefore the equivalent resistance is just the resistor itself, namely 1k Ohm.

4. Calculate the time constant.

5. The problem states that before time $t=0$ seconds, the capacitor voltage is zero. Therefore, the instant before time $t=0$, the capacitor voltage must be zero. Since the capacitor voltage cannot change instantaneously, the voltage at time $t=0$ seconds must also be zero volts.

6. The capacitor is (after a long time) trying to get to 5 volts.

7. Solve the two simultaneous equations for $A$ and $B$. Although the algebra is not shown here, you must show your algebra.

8. Complete solution, boxed.

Sketch.
1. Solve for the capacitor voltage for all time for the following circuits. Switches open and/or close when indicated. Give a complete solution. Show all of your work. Sketch the capacitor voltage as indicated. Use numeric values when labeling tick marks. Each problem compares a single component difference. **Begin each part on a separate page.**

   **a. Different Battery Voltage** The switch closes at time $t=0$ seconds for both circuits. The initial capacitor voltage is zero volts for both circuits. Sketch $v_c(t)$ for both problems on the **same axes**. Concisely describe the differences and similarities between the circuits, and how the capacitor voltage and time to charge are affected.

   ![Diagram 1](image1)

   i.  

   ii.  

   **b. Different Initial Capacitor Voltage** The switch closes at time $t=0$ seconds for both circuits. The initial capacitor voltage is zero volts for circuit 1. The initial capacitor voltage is -5 volts for circuit 2. Sketch $v_c(t)$ for both problems on the **same axes**. Concisely describe the differences and similarities between the circuits, and how the capacitor voltage and time to charge are affected.

   ![Diagram 2](image2)

   i.  

   ii.  

   **c. Different Resistance** The switch closes at time $t=0$ seconds for both circuits. The initial capacitor voltage is 5 volts for both circuits. Sketch $v_c(t)$ for both problems on the **same axes**. Concisely describe the differences and similarities between the circuits, and how the capacitor voltage and time to charge are affected.

   ![Diagram 3](image3)

   i.  

   ii.  

   **d. Different Capacitance** The switch closes at time $t=0$ seconds for both circuits. The initial capacitor voltage is 1 volt for both circuits. Sketch $v_c(t)$ for both problems on the **same axes**. Concisely describe the differences and similarities between the circuits, and how the capacitor voltage and time to charge are affected.

   ![Diagram 4](image4)

   i.  

   ii.
e. **Charge and Discharge** The switch closes at time $t=0$ seconds for both circuits. The initial capacitor voltage is 4 volts for both circuits. Sketch $v_c(t)$ for both problems on the **same axes**. Concisely describe the differences and similarities between the circuits, and how the capacitor voltage and time to charge are affected.

2. Solve for the capacitor voltage for all time. The switch switches from D to E at time $t=0s$. Give a complete solution. Show all of your work. Sketch a graph of the capacitor voltage from $-\tau \leq t \leq 5\tau$. The switch is connected to D for a *long* time. The switch switches to the battery at time $t=0$ seconds.

3. Derive two mathematical equations that relate a capacitor's initial ($v_{CI}$) and final voltage ($v_{CF}$) to variables $A$ and $B$ from the general solution in the following form:
   - $A = \text{Derive an expression}$
   - $B = \text{Derive an expression}$

4. Solve for $V_p$ and give in simplest form. Show your work.
   $$\frac{V_p - V_a}{R_a} + \frac{V_p - V_b}{R_b} + \frac{V_p - V_c}{R_c} = 0$$

5. Show (formal mathematical proof) that the following equation
   $$v_c(t) = A + Be^{-\frac{t}{RC}}$$
   is a solution to the following differential equation developed in class.
   $$C \frac{d}{dt} \left[ \frac{v_c(t)}{R} \right] + \frac{v_c(t)}{R} \frac{V}{R} = 0$$